12-8 Translations of Trigonometric Graphs

State the amplitude, period, and phase shift for each function. Then graph the function.

1. $y = \sin(\theta - 180°)$

SOLUTION:

Given $a = 1$, $b = 1$ and $h = 180°$.

Amplitude:

$$|a| = |1| \\ = 1$$

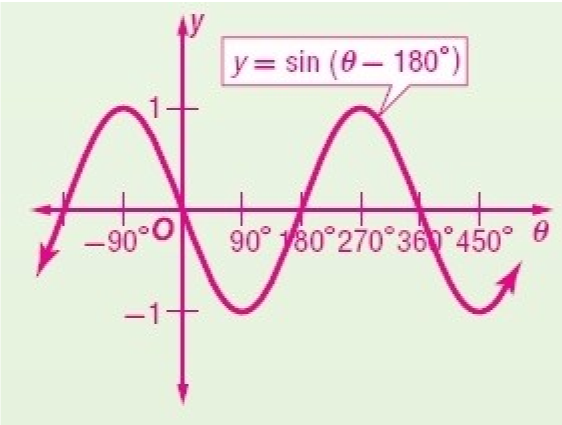
Period:

$$\frac{360°}{|b|} = \frac{360°}{|1|} \\ = 360°$$

Phase shift:

$$h = 180°$$

Graph $y = \sin\theta$ shifted $180°$ to the right.



2. $y = \tan\left(\theta - \frac{\pi}{4}\right)$

SOLUTION:

Given b = 1 and $h = \frac{\pi}{4}$.

Amplitude:
No amplitude

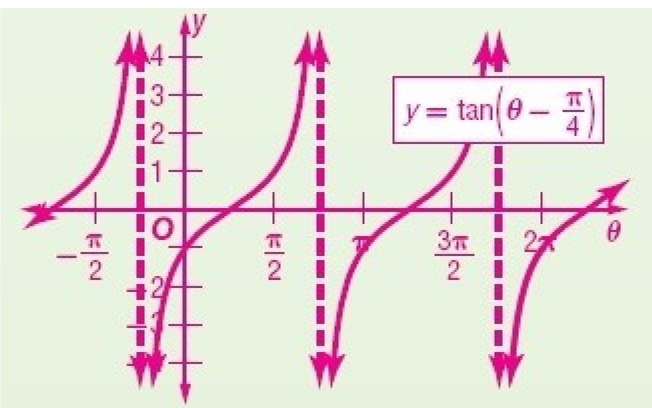
Period:

$$\frac{180^\circ}{|b|} = \frac{180^\circ}{|1|}$$
$$= 180^\circ$$

Phase shift:

$$h = \frac{\pi}{4}$$

Graph $y = \tan\theta$ shifted $\frac{\pi}{4}$ units to the right.



3. $y = \sin\left(\theta - \frac{\pi}{2}\right)$

SOLUTION:

Given a = 1, b = 1 and $h = \frac{\pi}{2}$.

Amplitude:

$$\begin{aligned}|a| &= |1|\\ &= 1\end{aligned}$$

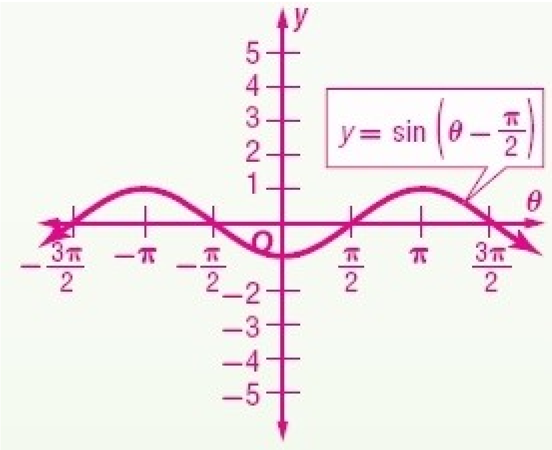
Period:

$$\begin{aligned}\frac{2\pi}{|b|} &= \frac{2\pi}{|1|}\\ &= 2\pi\end{aligned}$$

Phase shift:

$$h = \frac{\pi}{2}$$

Graph $y = \sin\theta$ shifted $\frac{\pi}{2}$ units to the right.



4. $y=\frac{1}{2}\cos(\theta+90°)$

SOLUTION:

Given $a = \frac{1}{2}$, $b = 1$ and $h = -90°$.

Amplitude:

$$|a| = \left|\frac{1}{2}\right| \\ = \frac{1}{2}$$

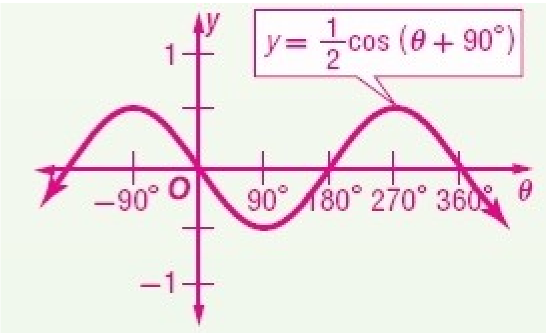
Period:

$$\frac{360°}{|b|} = \frac{360°}{|1|} \\ = 360°$$

Phase shift:

$$h = -90°$$

Graph $y=\frac{1}{2}\cos\theta$ shifted $90°$ to the left.



12-8 Translations of Trigonometric Graphs

State the amplitude, period, vertical shift, and equation of the midline for each function. Then graph the function.

5. $y = \cos \theta + 4$

SOLUTION:

Given $a = 1$, $b = 1$ and $k = 4$.

Amplitude:

$$|a| = |1|$$
$$= 1$$

Period:

$$\frac{360^\circ}{|b|} = \frac{360^\circ}{|1|}$$
$$= 360^\circ$$

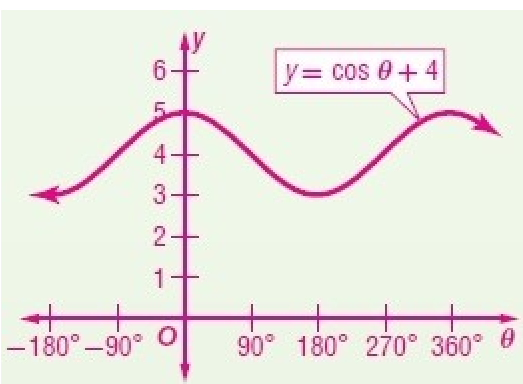
Vertical shift:

$k = 4$

Midline:

$y = 4$

To graph $y = \cos\theta + 4$, first draw the midline. Then use it to graph $y = \cos\theta$ shifted 4 units up.



6. $y = \sin \theta - 2$

SOLUTION:

The amplitude, period, vertical shift, and midline of the function $y = \sin\theta - 2$ is given by

Amplitude:

$$|a| = |1| \\ = 1$$

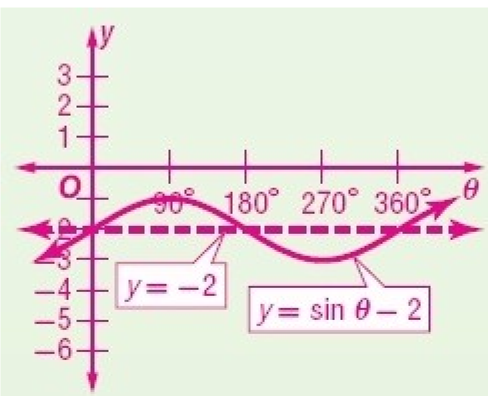
Period:

$$\frac{360^\circ}{|b|} = \frac{360^\circ}{|1|} \\ = 360^\circ$$

Vertical shift: $k = -2$

Midline: $y = -2$

To graph $y = \sin\theta - 2$, first draw the midline. Then use it to graph $y = \sin\theta$ shifted 2 units down.



7. $y = \frac{1}{2}\tan\theta + 1$

SOLUTION:

Given b = 1 and k = 1.

Amplitude:
No amplitude

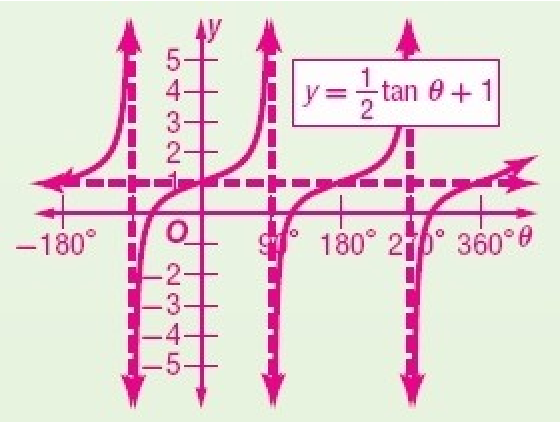
Period:

$$\frac{180^\circ}{|b|} = \frac{180^\circ}{|1|}$$
$$= 180^\circ$$

Vertical shift:
$k = 1$

Midline:
$y = 1$

To graph $y = \frac{1}{2}\tan\theta + 1$, first draw the midline. Then use it to graph $y = \frac{1}{2}\tan\theta$ shifted 1 unit up.



8. $y = \sec \theta - 5$

SOLUTION:

Given $b = 1$ and $k = -5$.

Amplitude:
No amplitude

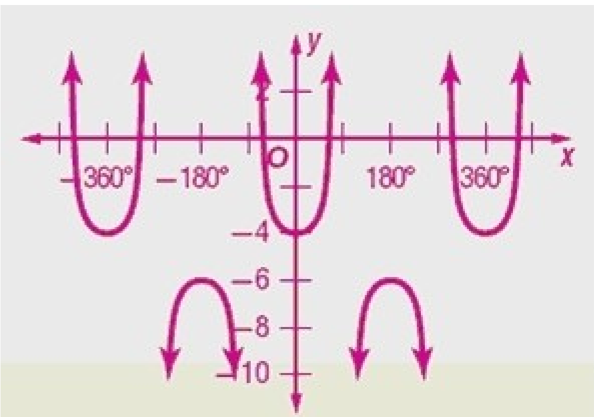
Period:

$$\frac{360^\circ}{|b|} = \frac{360^\circ}{|1|}$$
$$= 360^\circ$$

Vertical shift:
$k = -5$

Midline:
$y = -5$

First, graph the midline. Then graph $y = \sec \theta - 5$ using the midline as reference.



CCSS REGULARITY State the amplitude, period, phase shift, and vertical shift for each function. Then graph the function.

9. $y = 2 \sin (\theta + 45°) + 1$

SOLUTION:

Given $a = 2$, $b = 1$, $h = –45°$ and $k = 1$.

Amplitude:

$$\begin{aligned}|a| &= |2| \\ &= 2\end{aligned}$$

Period:

$$\begin{aligned}\frac{360°}{|b|} &= \frac{360°}{|1|} \\ &= 360°\end{aligned}$$

Phase shift:

$h = -45°$

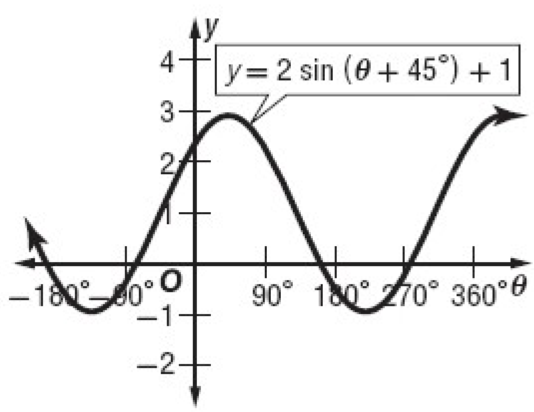
Vertical shift:

$k = 1$

Midline:

$y = 1$

First, graph the midline. Since the amplitude is 2, draw dashed line 2 units above and 2 units below the midline. Then graph $y = 2\sin\theta + 1$ using the midline as reference. Then shift the graph $45°$ to the left.



10. $y = \cos 3(\theta - \pi) - 4$

SOLUTION:

Given $a = 1$, $b = 3$, $h = \pi$ and $k = -4$.

Amplitude:

$$|a| = |1|$$
$$= 1$$

Period:

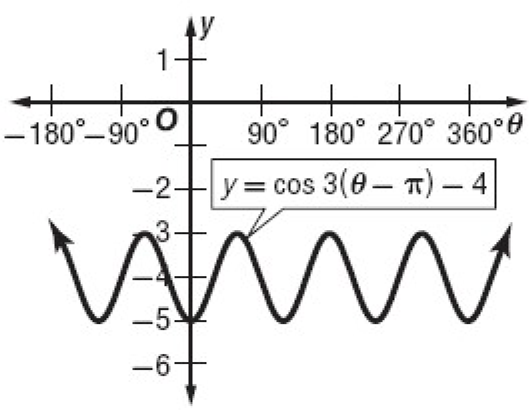
$$\frac{360^\circ}{|b|} = \frac{360^\circ}{|3|}$$
$$= 120^\circ$$

Phase shift: $h = \pi$

Vertical shift: $k = -4$

Midline: $y = -4$

First, graph the midline. Since the amplitude is 1, draw dashed line 1 units above and 1 unit below the midline. Then graph $y = \cos 3(\theta) - 4$ using the midline as reference. Then shift the graph π units to the right.



11. $y = \frac{1}{4}\tan 2(\theta + 30°) + 3$

SOLUTION:

Given $a = \frac{1}{4}$, $b = 2$, $h = -30°$ and $k = 3$.

Amplitude: No amplitude

Period:

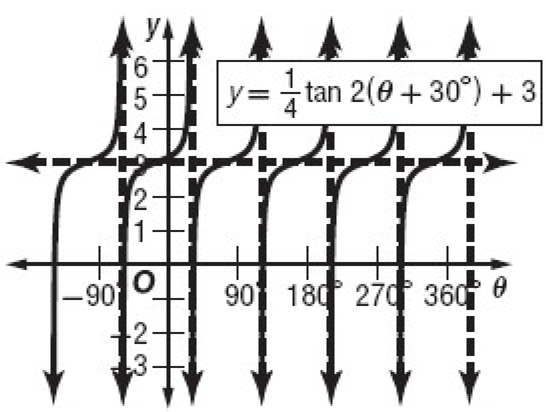
$$\frac{180^\circ}{|b|} = \frac{180^\circ}{|2|}$$
$$= 90^\circ$$

Phase shift: $h = -30^\circ$

Vertical shift: $k = 3$

Midline: $y = 3$

First, graph the midline. Then graph $y = \frac{1}{4}\tan 2(\theta) + 3$ using the midline as reference. Then shift the graph 30 units to the left.



12. $y = 4\sin\frac{1}{2}\left(\theta - \frac{\pi}{2}\right) + 5$

SOLUTION:

Given $a = 4$, $b = \frac{1}{2}$, $h = \frac{\pi}{2}$ and $k = 5$.

Amplitude:

$$\begin{aligned}|a| &= |4| \\ &= 4\end{aligned}$$

Period:

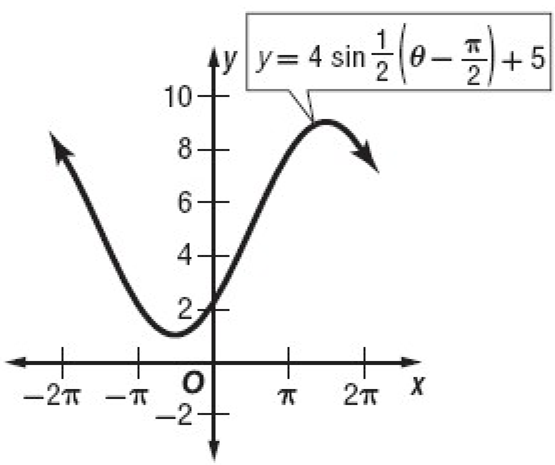
$$\begin{aligned}\frac{2\pi}{|b|} &= \frac{2\pi}{\left|\frac{1}{2}\right|} \\ &= 4\pi\end{aligned}$$

Phase shift: $h = \frac{\pi}{2}$

Vertical shift: $k = 5$

Midline: $y = 5$

First, graph the midline. Then graph $y = 4\sin\frac{1}{2}(\theta) + 5$ using the midline as reference. Then shift the graph $\frac{\pi}{2}$ units to the right.



13. **EXERCISE** While doing some moderate physical activity, a person's blood pressure oscillates between a maximum of 130 and a minimum of 90. The person's heart rate is 90 beats per minute. Write a sine function that represents the person's blood pressure *P* at time *t* seconds. Then graph the function.

SOLUTION:

Amplitude:

$$\begin{aligned}|a| &= |130-110|\\ &= |20|\\ &= 20\end{aligned}$$

Period:

Since the person's heart rate is 90 beats per minute, the heart beats every $\frac{60}{90}$ second. So, the period is $\frac{60}{90}$ second.

$$\begin{aligned}\frac{60}{90} &= \frac{2\pi}{|b|}\\ |b| &= \frac{180\pi}{60}\\ b &= 3\pi\end{aligned}$$

The midline lies halfway between the maximum and the minimum values.

$$\begin{aligned}y &= \frac{130+90}{2}\\ &= 110\end{aligned}$$

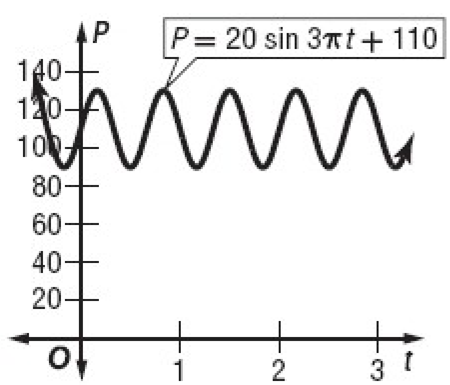
Therefore the vertical shift is $k = 110$.

Substitute 20 for *a*, 3π for *b*, 0 for *h*, and 110 for *k* in $h = a \sin b(t-h)+k$.

$$h = 20 \sin 3\pi(t-0)+110$$

$$h = 20 \sin 3\pi t + 110$$

Graph the function.



State the amplitude, period, and phase shift for each function. Then graph the function.

14. $y = \cos(\theta + 180°)$

SOLUTION:

Given $a = 1$, $b = 1$ and $h = -180°$.

Amplitude:

$$|a| = |1|$$
$$= 1$$

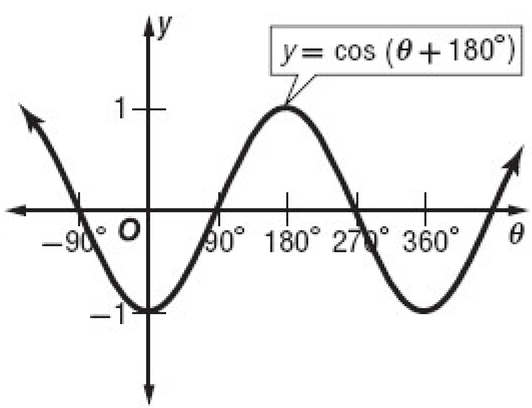
Period:

$$\frac{360°}{|b|} = \frac{360°}{|1|}$$
$$= 360°$$

Phase shift:

$$h = -180°$$

Graph $y = \cos\theta$ shifted $180°$ to the left.



15. $y = \tan(\theta - 90°)$

SOLUTION:
Given $b = 1$ and $h = 90°$.

Amplitude:
No amplitude

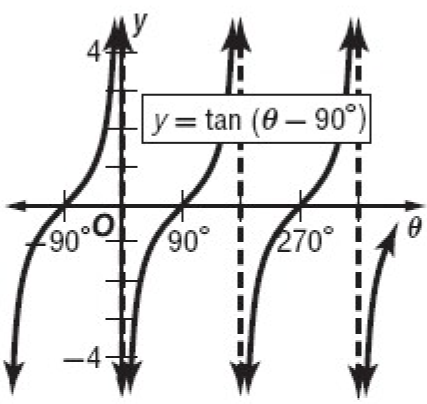
Period:

$$\frac{180°}{|b|} = \frac{180°}{|1|}$$
$$= 180°$$

Phase shift:

$$h = 90°$$

Graph $y = \tan\theta$ shifted $90°$ to the right.



16. $y = \sin(\theta + \pi)$

SOLUTION:

Given $a = 1$, $b = 1$ and $h = -\pi$.

Amplitude:

$$|a| = |1| \\ = 1$$

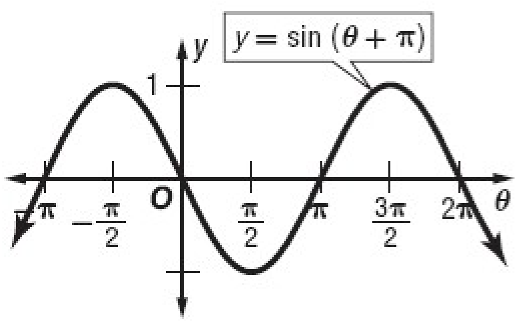
Period:

$$\frac{2\pi}{|b|} = \frac{2\pi}{|1|} \\ = 2\pi$$

Phase shift:

$h = -\pi$

Graph $y = \sin\theta$ shifted π units to the left.



12-8 Translations of Trigonometric Graphs

17. $y = 2\sin\left(\theta + \frac{\pi}{2}\right)$

SOLUTION:

Given a = 2, b = 1 and $h = -\frac{\pi}{2}$.

Amplitude:

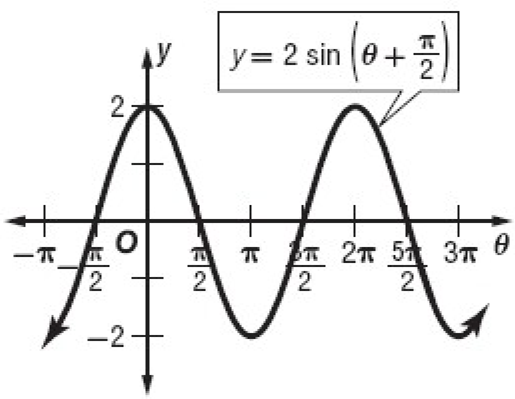
$$\begin{aligned}|a| &= |2| \\ &= 2\end{aligned}$$

Period:

$$\begin{aligned}\frac{2\pi}{|b|} &= \frac{2\pi}{|1|} \\ &= 2\pi\end{aligned}$$

Phase shift: $h = -\frac{\pi}{2}$

Graph $y = 2\sin\theta$ shifted $\frac{\pi}{2}$ units to the left.



18. $y = \tan\frac{1}{2}(\theta + 30°)$

SOLUTION:

Given $b = \frac{1}{2}$ and $h = –30°$.

Amplitude:
No amplitude

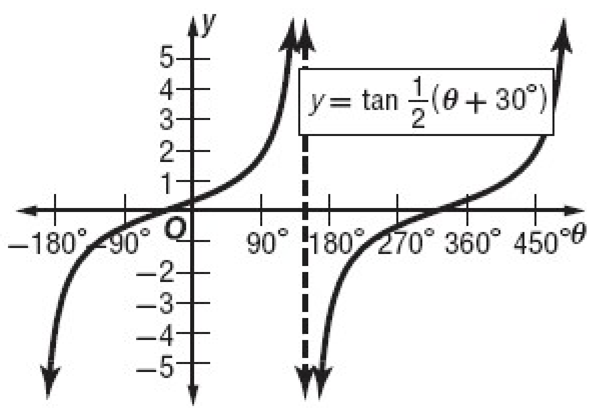
Period:

$$\frac{180^\circ}{|b|} = \frac{180^\circ}{\left|\frac{1}{2}\right|}$$
$$= 360^\circ$$

Phase shift:

$h = -30^\circ$

Graph $y = \tan\theta$ shifted 30° to the left.



19. $y = 3\cos\left(\theta - \frac{\pi}{3}\right)$

SOLUTION:

Given a = 3, b = 1 and $h = \frac{\pi}{3}$.

Amplitude:

$$\begin{aligned}|a| &= |3| \\ &= 3\end{aligned}$$

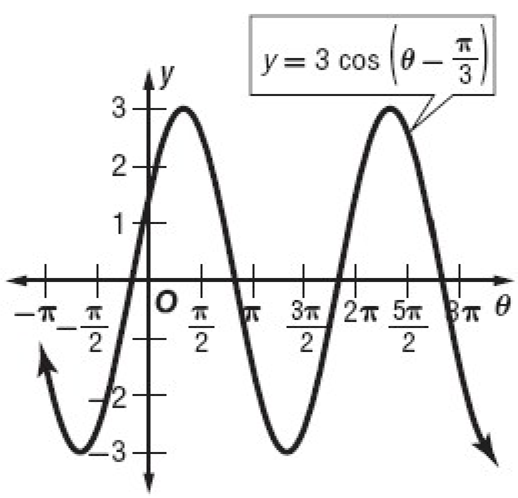
Period:

$$\begin{aligned}\frac{2\pi}{|b|} &= \frac{2\pi}{|1|} \\ &= 2\pi\end{aligned}$$

Phase shift:

$$h = \frac{\pi}{3}$$

Graph $y = 3\cos\theta$ shifted $\frac{\pi}{3}$ units to the right.



State the amplitude, period, vertical shift, and equation of the midline for each function. Then graph the function.

20. $y = \cos\theta + 3$

SOLUTION:

Given $a = 1$, $b = 1$ and $k = 3$.

Amplitude:

$$\begin{aligned}|a| &= |1| \\ &= 1\end{aligned}$$

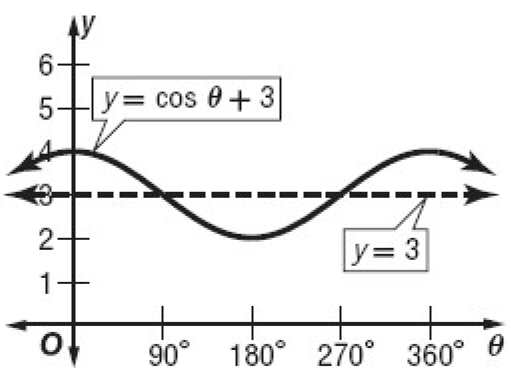
Period:

$$\begin{aligned}\frac{360^\circ}{|b|} &= \frac{360^\circ}{|1|} \\ &= 360^\circ\end{aligned}$$

Vertical shift: $k = 3$

Midline: $y = 3$

To graph $y = \cos\theta + 3$, first draw the midline. Then use it to graph $y = \cos\theta$ shifted 3 units up.



21. $y = \tan\theta - 1$

SOLUTION:
Given $a = 1$, $b = 1$ and $k = -1$.

Amplitude: No amplitude

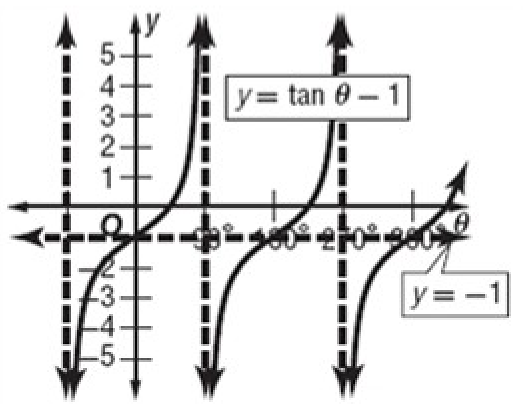
Period:

$$\frac{180^\circ}{|b|} = \frac{180^\circ}{|1|}$$
$$= 180^\circ$$

Vertical shift: $k = -1$

Midline: $y = -1$

To graph $y = \tan\theta - 1$, first draw the midline. Then use it to graph $y = \tan\theta$ shifted 1 unit down.



22. $y = \tan\theta + \frac{1}{2}$

SOLUTION:

Given a = 1, b = 1 and k = $\frac{1}{2}$.

Amplitude: No amplitude

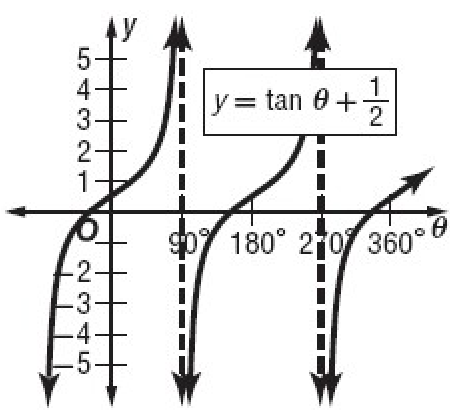
Period:

$$\frac{180^\circ}{|b|} = \frac{180^\circ}{|1|}$$
$$= 180^\circ$$

Vertical shift: $k = \frac{1}{2}$

Midline: $y = \frac{1}{2}$

To graph $y = \tan\theta + \frac{1}{2}$, first draw the midline. Then use it to graph $y = \tan\theta$ shifted $\frac{1}{2}$ units up.



23. $y = 2 \cos \theta - 5$

SOLUTION:

Given $a = 2$, $b = 1$ and $k = -5$.

Amplitude:

$$\begin{aligned} |a| &= |2| \\ &= 2 \end{aligned}$$

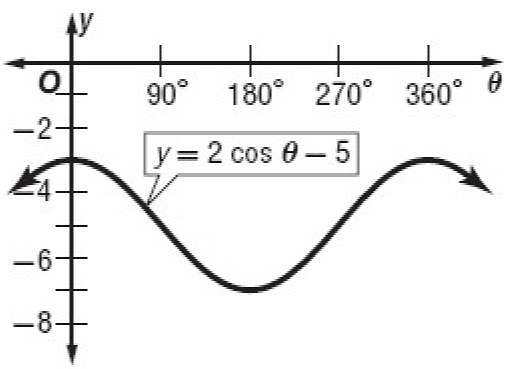
Period:

$$\begin{aligned} \frac{360^\circ}{|b|} &= \frac{360^\circ}{|1|} \\ &= 360^\circ \end{aligned}$$

Vertical shift: $k = -5$

Midline: $y = -5$

To graph $y = 2\cos\theta - 5$, first draw the midline. Then use it to graph $y = 2\cos\theta$ shifted 5 units down.



24. $y = 2 \sin \theta - 4$

SOLUTION:

Given $a = 2$, $b = 1$ and $k = -4$.

Amplitude:

$$\begin{aligned}|a| &= |2| \\ &= 2\end{aligned}$$

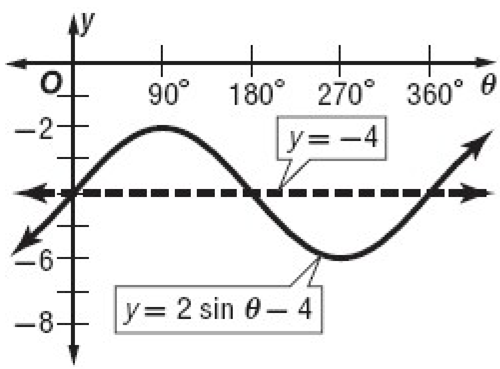
Period:

$$\begin{aligned}\frac{360^\circ}{|b|} &= \frac{360^\circ}{|1|} \\ &= 360^\circ\end{aligned}$$

Vertical shift: $k = -4$

Midline: $y = -4$

To graph $y = 2\sin\theta - 4$, first draw the midline. Then use it to graph $y = 2\sin\theta$ shifted 4 units down.



25. $y = \frac{1}{3}\sin\theta + 7$

SOLUTION:

Given $a = \frac{1}{3}$, $b = 1$ and $k = 7$.

Amplitude:

$$|a| = \left|\frac{1}{3}\right|$$
$$= \frac{1}{3}$$

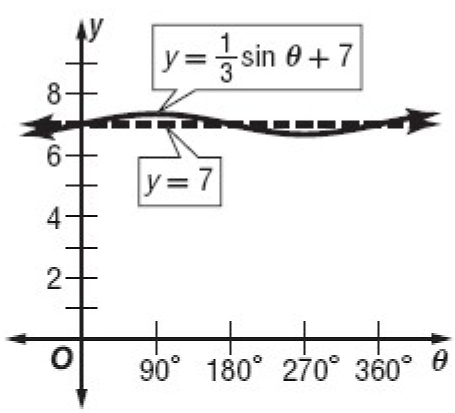
Period:

$$\frac{360^\circ}{|b|} = \frac{360^\circ}{|1|}$$
$$= 360^\circ$$

Vertical shift: $k = 7$

Midline: $y = 7$

To graph $y = \frac{1}{3}\sin\theta + 7$, first draw the midline. Then use it to graph $y = \frac{1}{3}\sin\theta$ shifted 7 units up.



State the amplitude, period, phase shift, and vertical shift for each function. Then graph the function.

26. $y = 4\sin(\theta - 60^\circ) - 1$

SOLUTION:

Given a = 4, b = 1, h = 60° and k = –1.

Amplitude:

$$\begin{aligned} |a| &= |4| \\ &= 4 \end{aligned}$$

Period:

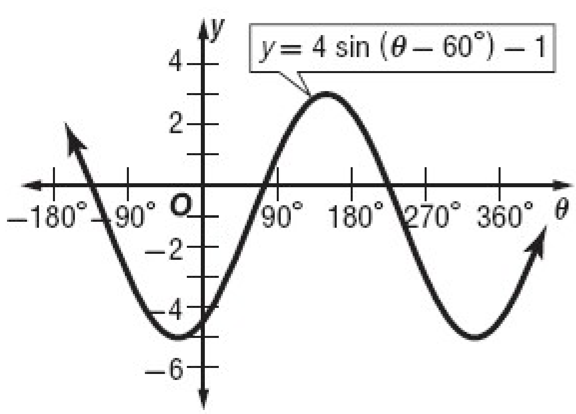
$$\begin{aligned} \frac{360^\circ}{|b|} &= \frac{360^\circ}{|1|} \\ &= 360^\circ \end{aligned}$$

Phase shift: $h = 60^\circ$

Vertical shift: $k = -1$

Midline: $y = -1$

First, graph the midline. Then graph $y = 4\sin\theta - 1$ using the midline as reference. Then shift the graph 60° to the right.



27. $y = \cos \frac{1}{2}(\theta - 90°) + 2$

SOLUTION:

Given a = 1, b = $\frac{1}{2}$, h = 90° and k = 2.

Amplitude:

$$\begin{aligned} |a| &= |1| \\ &= 1 \end{aligned}$$

Period:

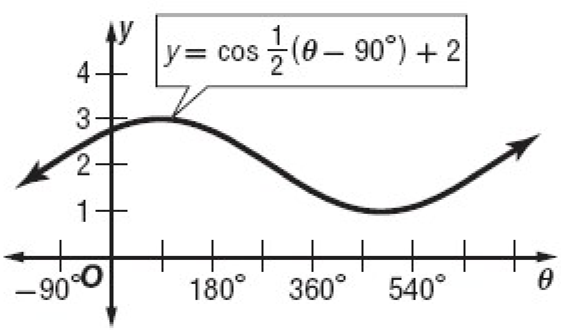
$$\begin{aligned} \frac{360^\circ}{|b|} &= \frac{360^\circ}{\left|\frac{1}{2}\right|} \\ &= 720^\circ \end{aligned}$$

Phase shift: $h = 90^\circ$

Vertical shift: $k = 2$

Midline: $y = 2$

First, graph the midline. Then graph $y = \cos \frac{1}{2}(\theta) + 2$ using the midline as reference. Then shift the graph 90° to the right.



28. $y = \tan(\theta + 30°) - 2$

SOLUTION:

Given $a = 1$, $b = 1$, $h = -30°$ and $k = -2$.

Amplitude: No amplitude

Period:

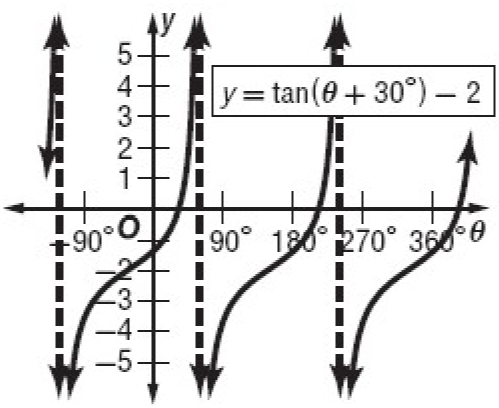
$$\frac{180°}{|b|} = \frac{180°}{|1|}$$
$$= 180°$$

Phase shift: $h = -30°$

Vertical shift: $k = -2$

Midline: $y = -2$

First, graph the midline. Then graph $y = \tan(\theta) - 2$ using the midline as reference. Then shift the graph $30°$ to the left.



29. $y = 2\tan 2\left(\theta + \frac{\pi}{4}\right) - 5$

SOLUTION:

Given a = 2, b = 2, h = $\frac{\pi}{4}$ and k = –5.

Amplitude: No amplitude

Period:

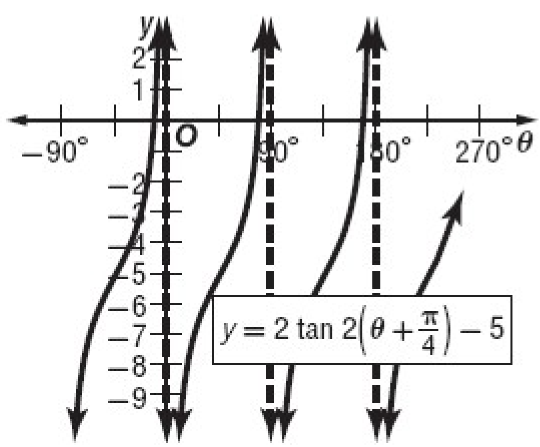
$$\frac{\pi}{|b|} = \frac{\pi}{|2|}$$
$$= \frac{\pi}{2}$$

Phase shift: $h = -\frac{\pi}{4}$

Vertical shift: $k = -5$

Midline: $y = -5$

First, graph the midline. Then graph $y = 2\tan 2(\theta) - 5$ using the midline as reference. Then shift the graph $\frac{\pi}{4}$ units to the left.



30. $y = \frac{1}{2}\sin\left(\theta - \frac{\pi}{2}\right) + 4$

SOLUTION:

Given $a = \frac{1}{2}$, $b = 1$, $h = \frac{\pi}{2}$ and $k = 4$.

Amplitude:

$$|a| = \left|\frac{1}{2}\right| = \frac{1}{2}$$

Period:

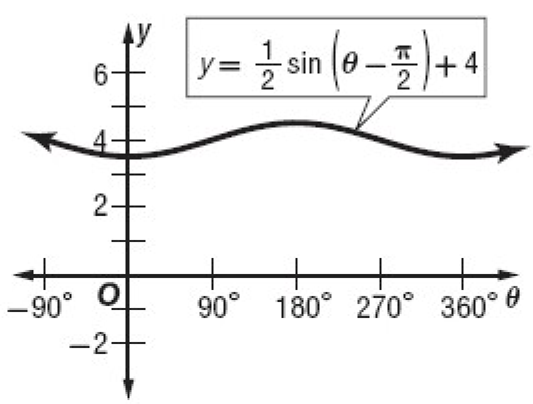
$$\frac{2\pi}{|b|} = \frac{2\pi}{|1|} = 2\pi$$

Phase shift: $h = \frac{\pi}{2}$

Vertical shift: $k = 4$

Midline: $y = 4$

First, graph the midline. Then graph $y = \frac{1}{2}\sin\theta + 4$ using the midline as reference. Then shift the graph $\frac{\pi}{2}$ units to the right.



31. $y = \cos 3(\theta - 45°) + \frac{1}{2}$

SOLUTION:

Given a = 1, b = 3, h = 45° and k = $\frac{1}{2}$.

Amplitude:

$$|a| = |1|$$
$$= 1$$

Period:

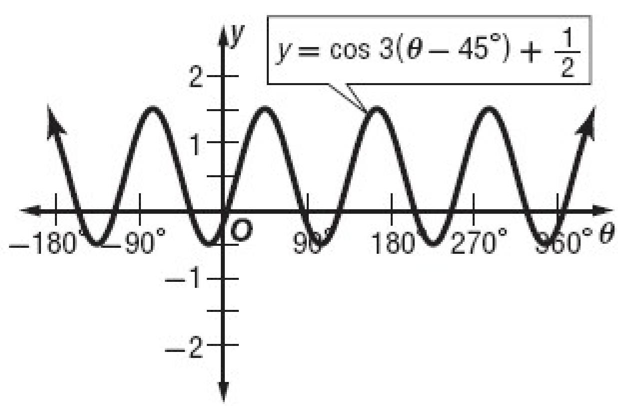
$$\frac{360°}{|b|} = \frac{360°}{|3|}$$
$$= 120°$$

Phase shift: $h = 45°$

Vertical shift: $k = \frac{1}{2}$

Midline: $y = \frac{1}{2}$

First, graph the midline. Then graph $y = \cos 3(\theta) + \frac{1}{2}$ using the midline as reference. Then shift the graph $45°$ to the right.



32. $y = 3 + 5 \sin 2(\theta - \pi)$

SOLUTION:

Given $a = 5$, $b = 2$, $h = \pi$ and $k = 3$.

Amplitude:

$$\begin{aligned}|a| &= |5| \\ &= 5\end{aligned}$$

Period:

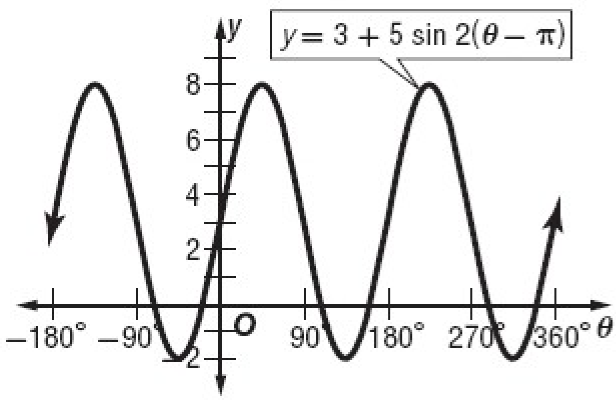
$$\begin{aligned}\frac{2\pi}{|b|} &= \frac{2\pi}{|2|} \\ &= \pi\end{aligned}$$

Phase shift: $h = \pi$

Vertical shift: $k = 3$

Midline: $y = 3$

First, graph the midline. Then graph $y = 3 + 5\sin 2\theta$ using the midline as reference. Then shift the graph π units to the right.



33. $y = -2 + 3\sin\frac{1}{3}\left(\theta - \frac{\pi}{2}\right)$

SOLUTION:

Given a = 3, b = $\frac{1}{3}$, h = $\frac{\pi}{2}$ and k = –2.

Amplitude:

$$\begin{aligned}|a| &= |3|\\ &= 3\end{aligned}$$

Period:

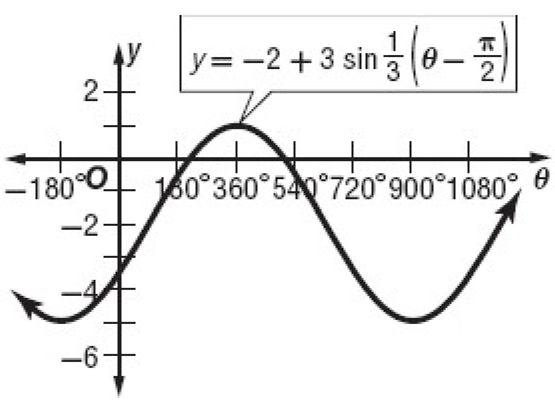
$$\begin{aligned}\frac{2\pi}{|b|} &= \frac{2\pi}{\left|\frac{1}{3}\right|}\\ &= 6\pi\end{aligned}$$

Phase shift: $h = \frac{\pi}{2}$

Vertical shift: $k = -2$

Midline: $y = -2$

First, graph the midline. Then graph $y = 3\sin\frac{1}{3}(\theta) - 2$ using the midline as reference. Then shift the graph $\frac{\pi}{2}$ units to the right.



34. **TIDES** The height of the water in a harbor rose to a maximum height of 15 feet at 6:00 p.m. and then dropped to a minimum level of 3 feet by 3:00 a.m. The water level can be modeled by the sine function. Write an equation that represents the height *h* of the water *t* hours after noon on the first day.

SOLUTION:

The maximum and the minimum height is 15ft and 3 ft respectively.

Therefore, the amplitude is $\left|\frac{15-3}{2}\right|$ or 6.

The time taken for half cycle is 9 hrs. Therefore, the period is 18 hrs.
Find the value of b.

$$18 = \frac{2\pi}{|b|}$$
$$b = \pm\frac{\pi}{9}$$

Since the period of the function is 18 hrs, one fourth of the period is 4.5 hrs.
Therefore, the horizontal shift is 6 – 4.5 or 1.5.
That is, h = 1.5.
The vertical shift is $\frac{15+3}{2}$ or 9.

That is k = 9.
Substitute the values of a, b, $h$and k in the standard equation of the sine function.

$$h = 6\sin\left[\frac{\pi}{9}(t-1.5)\right]+9$$

35. **LAKES** A buoy marking the swimming area in a lake oscillates each time a speed boat goes by. Its distance *d* in feet from the bottom of the lake is given by $d = 1.8\sin\frac{3\pi}{4}t + 12$, where *t* is the time in seconds.

Graph the function. Describe the minimum and maximum distances of the buoy from the bottom of the lake when a boat passes by.

SOLUTION:

Given a = 1.8, b = $\frac{3\pi}{4}$, h = 0 and k = 12.

Amplitude:

$$|a| = |1.8|$$
$$= 1.8$$

Period:

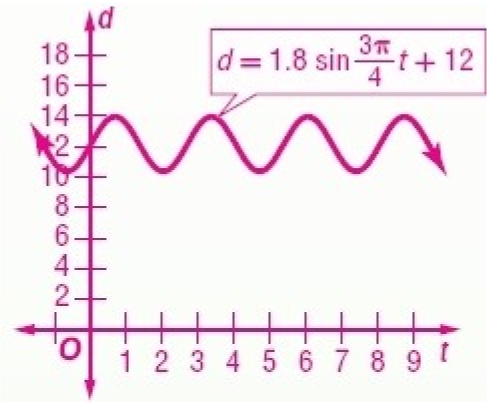
$$\frac{2\pi}{|b|} = \frac{2\pi}{\left|\frac{3\pi}{4}\right|}$$
$$= \frac{8}{3}$$

Phase shift: No phase shift

Vertical shift: $k = 12$

Midline: $y = 12$

First, graph the midline. Then graph $d = 1.8\sin\frac{3\pi}{4}t + 12$ using the midline as reference.



Since the maximum value is the value of the midline plus the amplitude, the maximum distance is $d = 1.8 + 12$ or 13.8.
Since the minimum value is the value of the midline minus the amplitude, the minimum distance is $d = 12 - 1.8$ or 10.2

36. **FERRIS WHEEL** Suppose a Ferris wheel has a diameter of approximately 520 feet and makes one complete revolution in 30 minutes. Suppose the lowest car on the Ferris wheel is 5 feet from the ground. Let the height at the top of the wheel represent the height at time 0. Write an equation for the height of a car *h* as a function of time *t*. Then graph the function.

SOLUTION:

The midline lies halfway between the maximum and the minimum values $y = \frac{525 + 5}{2}$ or 265

Therefore the vertical shift is $k = 265$.
Amplitude:
$$|a| = \left|\frac{525 - 5}{2}\right|$$
$$= 260$$

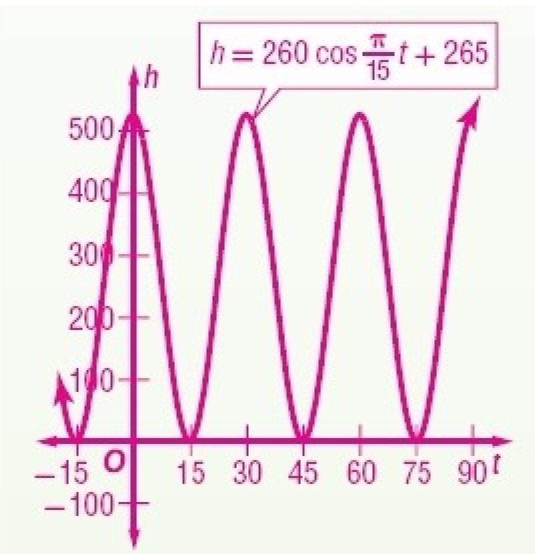
Period:
Since the wheel makes one complete revolution in 30 minutes, the period is 30 minutes.

$$30 = \frac{2\pi}{|b|}$$
$$|b| = \frac{2\pi}{30}$$
$$b = \frac{\pi}{15}$$

Substitute 260 for *a*, $\frac{\pi}{15}$ for *b*, 265 for *t* in $h = a \sin b(t - h) + k$.

$$h = 260 \sin \frac{\pi}{15}(t - 0) + 265$$
$$h = 260 \sin \frac{\pi}{15}t + 265$$

Graph the function.



12-8 Translations of Trigonometric Graphs

Write an equation for each translation.

37. $y = \sin x$, 4 units to the right and 3 units up

SOLUTION:

The sine function involving phase shifts and vertical shifts is $y = a\sin b(x - h) + k$.

Given $a = 1, b = 1, h = 4, k = 3$.

Therefore, the equation is $y = \sin(x - 4) + 3$.

38. $y = \cos x$, 5 units to the left and 2 units down

SOLUTION:

The cosine function involving phase shifts and vertical shifts is $y = a\cos b(x - h) + k$.

Given $a = 1, b = 1, h = -5, k = -2$.

Therefore, the equation is $y = \cos(x + 5) - 2$.

39. $y = \tan x$, π units to the right and 2.5 units up

SOLUTION:

The tangent function involving phase shifts and vertical shifts is $y = a\tan b(x - h) + k$.

Given $a = 1, b = 1, h = \pi, k = 2.5$.

Therefore, the equation is $y = \tan(x - \pi) + 2.5$

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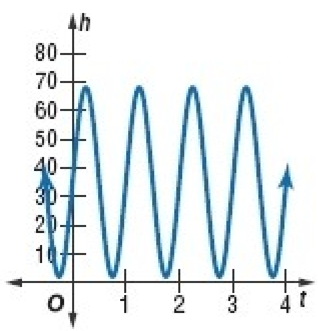
40. **JUMP ROPE** The graph approximates the height of a jump rope h in inches as a function of time t in seconds. A maximum point on the graph is (1.25, 68), and a minimum point is (2.75, 2).



a. Describe what the maximum and minimum points mean in the context of the situation.

b. What is the equation for the midline, the amplitude, and the period of the function?

c. Write an equation for the function.

SOLUTION:

a. At 1.25 seconds, the height of the rope is 68 inches and at 2.75 seconds, the height of the rope is 2 inches.

b. The midline lies halfway between the maximum and the minimum values.

$$\begin{aligned} y &= \frac{68+2}{2} \\ &= 35 \end{aligned}$$

Therefore the vertical shift is $k = 35$.

Midline: $y = 35$

Amplitude:

$$\begin{aligned} |a| &= \left|\frac{68-2}{2}\right| \\ &= 33 \end{aligned}$$

The graph completes 1.5 cycles in 1.5 seconds (between 1.25 and 2.75).

Therefore, period is 1

.

c. Find the value of *b*.

$$\begin{aligned} \text{Period} &= \frac{2\pi}{|b|} \\ 1 &= \frac{2\pi}{|b|} \\ b &= \pm 2\pi \end{aligned}$$

Substitute 33 for *a*, 2π for *b*, 35 for *k* in $h = a \sin b(t-h) + k$.

$$h = 33 \sin (t-0) + 35$$
$$h = 33 \sin 2\pi t + 35$$

41. **CAROUSEL** A horse on a carousel goes up and down 3 times as the carousel makes one complete rotation. The maximum height of the horse is 55 inches, and the minimum height is 37 inches. The carousel rotates once every 21 seconds. Assume that the horse starts and stops at its median height.

a. Write an equation to represent the height of the horse *h* as a function of time *t* seconds.

b. Graph the function.

c. Use your graph to estimate the height of the horse after 8 seconds. Then use a calculator to find the height to the nearest tenth.

SOLUTION:

a. Amplitude:

$$\begin{aligned}|a| &= |55-46| \\ &= |9| \\ &= 9\end{aligned}$$

Since the carousel rotates once every 21 seconds, and a horse on the carousel goes up and down three times in one rotation, the time taken for the horse to go up and down once is 7 seconds. So, the period is 7 seconds.

Find the value of b.

$$7 = \frac{2\pi}{|b|}$$

$$|b| = \frac{2\pi}{7}$$

$$b = \frac{2\pi}{7}$$

The midline lies halfway between the maximum and the minimum values.

$$\begin{aligned}y &= \frac{55+37}{2} \\ &= 46\end{aligned}$$

Therefore the vertical shift is $k = 46$.

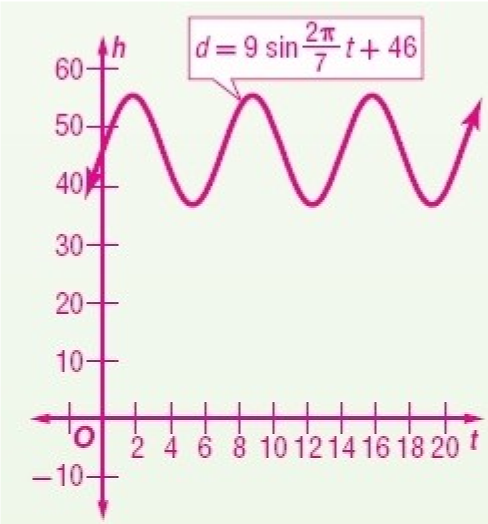
Midline: $y = 46$

Substitute 9 for a, $\frac{2\pi}{7}$ for b, 0 for h, and 46 for k in $h = a\sin b(t-h)+k$.

$$h = 9\sin\frac{2\pi}{7}(t-0)+46$$

$$h = 9\sin\frac{2\pi}{7}t+46$$

b. Graph the function.



c. Sample answer:
Substitute 8 for *t* to find the height.

$$h = 9\sin\left(\frac{2\pi}{7}\right)(8) + 46$$
$$= 9\sin\left(\frac{16\pi}{7}\right) + 46$$
$$\approx 7.03 + 46$$
$$\approx 53.0$$

Therefore the height of the horse after 8 seconds is about 53 inches.

42. **CCSS REASONING** During one month, the outside temperature fluctuates between 40°F and 50°F. A cosine curve approximates the change in temperature, with a high of 50°F being reached every four days.

a. Describe the amplitude, period, and midline of the function that approximates the temperature y on day d.

b. Write a cosine function to estimate the temperature y on day d.

c. Sketch a graph of the function.

d. Estimate the temperature on the 7th day of the month.

SOLUTION:

a. Amplitude:

$$|a| = \left|\frac{50 - 40}{2}\right|$$
$$= 5$$

Since the change in temperature with a high of 50° F being reached every four days, the period is 4.
The midline lies halfway between the maximum and the minimum values.

$$y = \frac{50 + 40}{2}$$
$$= 45$$

Therefore the vertical shift is $k = 45$.

Midline: $y = 45$

b. Find the value of b.

$$4 = \frac{2\pi}{|b|}$$
$$|b| = \frac{2\pi}{4}$$
$$b = \frac{\pi}{2}$$

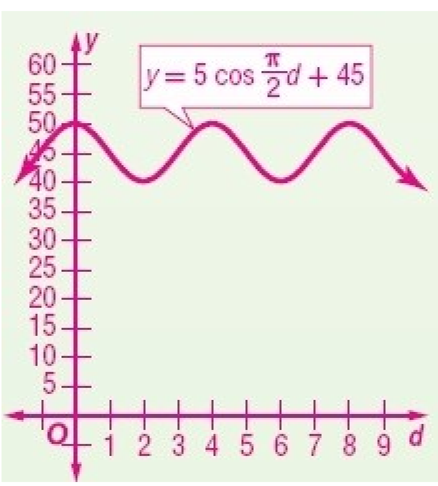
Write an equation for the function.

$$h = a \cos b(d - h) + k$$

Substitute 5 for a, $\frac{\pi}{2}$ for b, 0 for h, and 45 for k.

$$h = 5\cos\frac{\pi}{2}(d - 0) + 45$$
$$h = 5\cos\frac{\pi}{2}d + 45$$

c. Graph the function.



d. Substitute 7 for d to find the temperature.

$$\begin{aligned} h &= 5\cos\left(\frac{\pi}{2}\right)(7) + 45 \\ &= 5\cos\left(\frac{7\pi}{2}\right) + 45 \\ &= 0 + 45 \\ &= 45 \end{aligned}$$

Therefore, the temperature on the 7^{th} day of the month is about 45°F.

Find a coordinate that represents a maximum for each graph.

43. $y = -2\cos\left(x - \frac{\pi}{2}\right)$

SOLUTION:

Sample answer:

$$\begin{aligned} y &= -2\cos\left(x - \frac{\pi}{2}\right) \\ \frac{-y}{2} &= \cos\left(x - \frac{\pi}{2}\right) \end{aligned}$$

The range of $\cos\left(x - \frac{\pi}{2}\right)$ is $-1 \le \frac{-y}{2} \le 1$.

$$\begin{aligned} &-1 \le \frac{-y}{2} \le 1 \\ &-2 \le -y \le 2 \\ &2 \ge y \ge -2 \end{aligned}$$

Substitute 2 for y and solve for x.

$$\begin{aligned} 2 &= -2\cos\left(x - \frac{\pi}{2}\right) \\ -1 &= \cos\left(x - \frac{\pi}{2}\right) \\ x - \frac{\pi}{2} &= \cos^{-1}(-1) \\ x &= \pi + \frac{\pi}{2} \text{ or } \frac{3\pi}{2} \end{aligned}$$

The coordinate of the maximum point is $\left(\frac{3\pi}{2}, 2\right)$.

44. $y = 4\sin\left(x + \frac{\pi}{3}\right)$

SOLUTION:

Sample answer:

$$y = 4\sin\left(x + \frac{\pi}{3}\right)$$
$$\frac{y}{4} = \sin\left(x + \frac{\pi}{3}\right)$$

The range of $\sin\left(x + \frac{\pi}{3}\right)$ is $-1 \le \frac{-y}{2} \le 1$.

$$-1 \le \frac{y}{4} \le 1$$
$$-4 \le y \le 4$$

Substitute 4 for y and solve for x.

$$4 = 4\sin\left(x + \frac{\pi}{3}\right)$$
$$1 = \sin\left(x + \frac{\pi}{3}\right)$$
$$x + \frac{\pi}{3} = \sin^{-1}(1)$$
$$x = \sin^{-1}(1) - \frac{\pi}{3}$$
$$x = \frac{\pi}{2} - \frac{\pi}{3} \text{ or } \frac{\pi}{6}$$

The coordinate of the maximum point is $\left(\frac{\pi}{6}, 4\right)$.

45. $y = 3\tan\left(x + \frac{\pi}{2}\right) + 2$

SOLUTION:

Since the amplitude is undefined for the tangent functions, there is no maximum value for $y = 3\tan\left(x + \frac{\pi}{2}\right) + 2$.

46. $y = -3\sin\left(x - \frac{\pi}{4}\right) - 4$

SOLUTION:

Sample answer:

$$y = -3\sin\left(x - \frac{\pi}{4}\right) - 4$$
$$-\frac{y+4}{3} = \sin\left(x - \frac{\pi}{4}\right)$$

The range of $\sin\left(x - \frac{\pi}{4}\right)$ is $-1 \le \frac{-(y+4)}{3} \le 1$.

$$-1 \le \frac{-(y+4)}{3} \le 1$$
$$-3 \le -(y+4) \le 3$$
$$1 \le -y \le 7$$
$$-7 \le y \le -1$$

Substitute –1 for y and solve for x.

$$-1 = -3\sin\left(x - \frac{\pi}{4}\right) - 4$$
$$3 = -3\sin\left(x - \frac{\pi}{4}\right)$$
$$x - \frac{\pi}{4} = \sin^{-1}(-1)$$
$$x = \sin^{-1}(-1) + \frac{\pi}{4}$$
$$= \frac{3\pi}{2} + \frac{\pi}{4} \text{ or } \frac{7\pi}{4}$$

The coordinate of the maximum point is $\left(\frac{7\pi}{4}, -1\right)$.

Compare each pair of graphs.

47. $y = -\cos 3\theta$ and $y = \sin 3(\theta - 90°)$

SOLUTION:

Given $a = -1$ and $b = 3$.

Amplitude:

$$|a| = |-1|$$
$$= 1$$

Period:

$$\frac{360^\circ}{|b|} = \frac{360^\circ}{|3|}$$
$$= 120^\circ$$

Draw the graph of $y = -\cos 3\theta$

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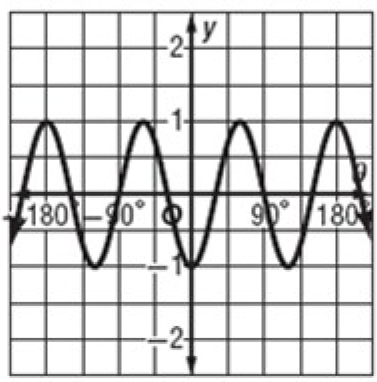


Given $a = -1$, $b = 3$ and $h = 90°$.

Amplitude:

$$|a| = |1|$$
$$= 1$$

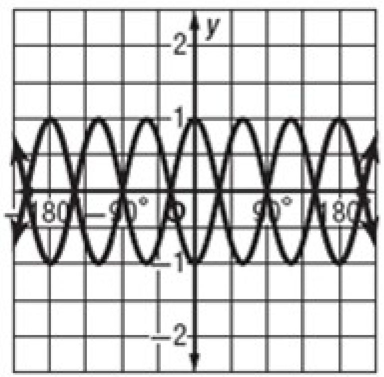
Period:

$$\frac{360^\circ}{|b|} = \frac{360^\circ}{|3|}$$
$$= 120^\circ$$

Phase shift:

$$h = 90^\circ$$

Graph $y = \sin 3\theta$ shifted 90° units to the right.



The graphs are reflections of each other over the *x*-axis.

48. $y = 2 + 0.5 \tan \theta$ and $y = 2 + 0.5 \tan (\theta + \pi)$

SOLUTION:

Given $a = 0.5$, $b = 1$, $h = 0$ and $k = 2$.

Amplitude: No amplitude

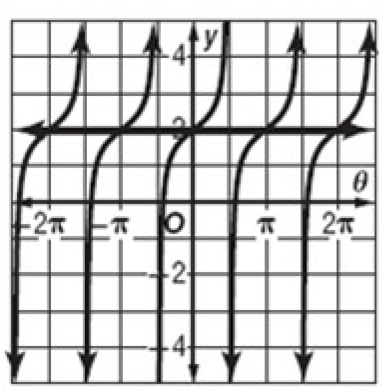
Period:

$$\frac{180^\circ}{|b|} = \frac{180^\circ}{|1|}$$
$$= 180^\circ$$

Vertical shift: $k = 2$

Midline: $y = 2$

To graph $y = 2 + 0.5 \tan \theta$, first draw the midline. Then use it to graph $y = 0.5 \tan \theta$ shifted 2 units up.



Given $a = 0.5$, $b = 1$, $h = \pi$ and $k = 2$.

Amplitude: No amplitude

Period:

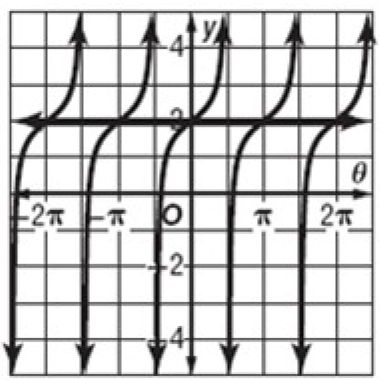
$$\frac{180^\circ}{|b|} = \frac{180^\circ}{|1|}$$
$$= 180^\circ$$

Phase shift: $h = -\pi$

Vertical shift: $k = 2$

Midline: $y = 2$

First, graph the midline. Then graph $y = 2 + 0.5\tan(\theta + \pi)$ using the midline as reference. Then shift the graph π units to the left.



Therefore, the graphs are identical.

49. $y = 2\sin\left(\theta - \frac{\pi}{6}\right)$ and $y = -2\sin\left(\theta + \frac{5\pi}{6}\right)$

SOLUTION:

Given a = 2, b = 1, h = $\frac{\pi}{6}$ and k = 0.

Amplitude:

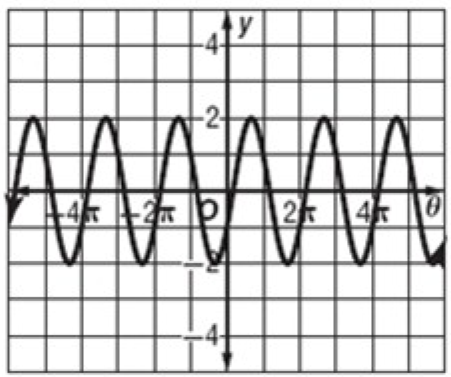
$$|a| = |2|$$
$$= 2$$

Period:

$$\frac{360^\circ}{|b|} = \frac{360^\circ}{|1|}$$
$$= 360^\circ$$

Phase shift: $h = \frac{\pi}{6}$

Graph $y = 2\sin\theta$ shifted $\frac{\pi}{6}$ units to the right.



Given a = –2, b = 1, h = $-\frac{5\pi}{6}$ and k = 0.

Amplitude:

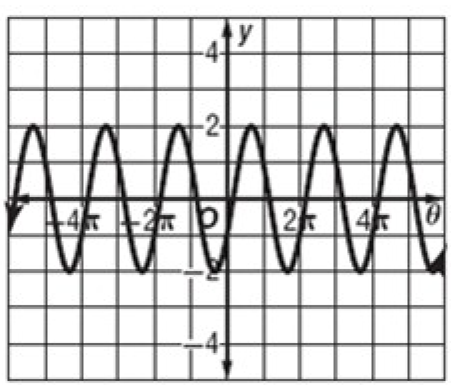
$$|a| = |-2|$$
$$= 2$$

Period:

$$\frac{360^\circ}{|b|} = \frac{360^\circ}{|1|}$$
$$= 360^\circ$$

Phase shift: $h = -\frac{5\pi}{6}$

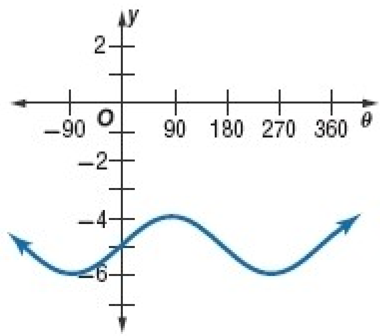
Graph $y = -2\sin\theta$ shifted $\frac{5\pi}{6}$ units to the left.



The graphs are identical.

Identify the period of each function. Then write an equation for the graph using the given trigonometric function.

50. sine



SOLUTION:

Period: $360°$

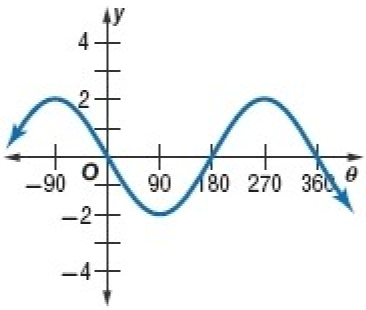
The function gets the maximum value of –4 and the minimum value of –6. Therefore, the amplitude is $\left|\frac{-6-(-4)}{2}\right|$ or 1 and the midline is at $y = \frac{-6-4}{2}$ or -5.

Since the period is 360°, $b = \frac{360°}{360°}$ or 1.

Since the midline is $y = -5$, the graph is shifted vertically down by 5 units. That is, $k = -5$.

Therefore the equation is $y = \sin\theta - 5$.

51. cosine



SOLUTION:

Period: 360°

The function gets the maximum value of 2 and the minimum value of –2. Therefore, the amplitude is $\left|\frac{-2-(2)}{2}\right|$ or 2 and the midline is at $y = \frac{-2+2}{2}$ or 0.

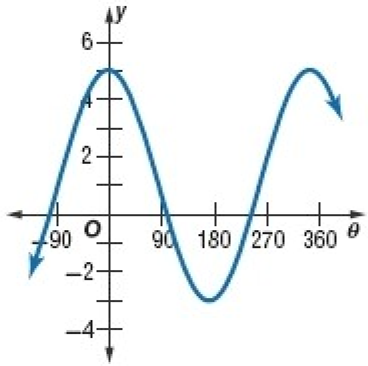
Since the period is 360°, $b = \frac{360^\circ}{360^\circ}$ or 1.

Since the function has shifted to the left by 90 units, h = –90°.

Since the midline is y = 0, there is no vertical shift. That is k = 0.

Therefore the equation is $y = 2\cos(\theta + 90^\circ)$.

52. cosine



SOLUTION:

Period: 360°

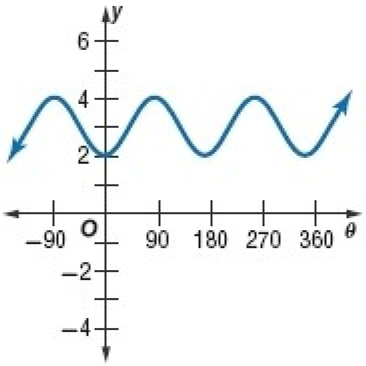
The function gets the maximum value of 5 and the minimum value of –3. Therefore, the amplitude is $\left|\frac{-3-(5)}{2}\right|$ or 4 and the midline is at $y = \frac{5-3}{2}$ or 1.

Since the period is 360°, $b=\frac{360^\circ}{360^\circ}$ or 1.

Since the midline is $y = 1$, graph is shifted vertically up by 1 unit. That is $k = 1$.

Therefore the equation is $y=4\cos(\theta)+1$.

53. sine



SOLUTION:

Period: 180°

The function gets the maximum value of 4 and the minimum value of 2. Therefore, the amplitude is $\left|\frac{4-2}{2}\right|$ or 1 and the midline is at $y = \frac{4+2}{2}$ or 3.

Since the period is 180°, $b = \frac{360^\circ}{180^\circ}$ or 2.

Since the function has shifted to the right by 45 units, $h = 45°$.

Since the midline is $y = 3$, graph is shifted vertically up by 1 unit. That is $k = 3$.

Therefore the equation is $y = \sin 2(\theta - 45^\circ) + 3$.

State the period, phase shift, and vertical shift. Then graph the function.

54. $y = \csc(\theta + \pi)$

SOLUTION:

Given $a = 1$, $b = 1$ and $h = -\pi$.

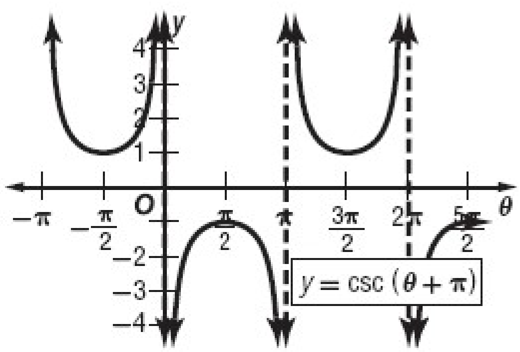
Period:

$$\frac{360^\circ}{|b|} = \frac{360^\circ}{|1|} = 360^\circ$$

Phase shift: $h = -\pi$

Vertical shift: No vertical shift

To graph $y = \csc(\theta + \pi)$, shift the graph of $y = \csc(\theta)$ to the left by π units.



55. $y = \cot \theta + 6$

SOLUTION:
Given $a = 1$, $b = 1$ and $k = 6$.

Period:

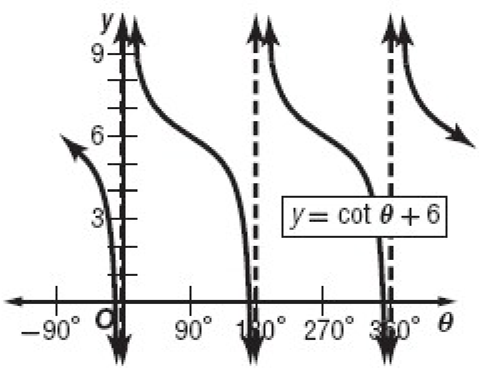
$$\frac{180^\circ}{|b|} = \frac{180^\circ}{|1|}$$
$$= 180^\circ$$

Phase shift: No phase shift.

Vertical shift: $k = 6$

Midline: $y = 6$

To graph $y = \csc(\theta) + 6$, first draw the midline. Then use it to graph $y = \cot\theta$ shifted 6 units up.



56. $y = \cot\left(\theta - \frac{\pi}{6}\right) - 2$

SOLUTION:

Given a = 1, b = 1, h = $\frac{\pi}{6}$ and k = –2.

Period:

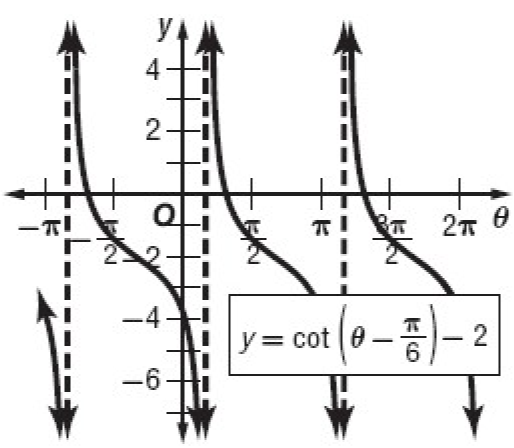
$$\frac{\pi}{|b|} = \frac{\pi}{|1|}$$
$$= \pi$$

Phase shift: $h = \frac{\pi}{6}$

Vertical shift: $k = -2$

Midline: $y = -2$

First, graph the midline. Then graph $y = \cot(\theta) - 2$ using the midline as reference. Then shift the graph $\frac{\pi}{6}$ units to the right.



57. $y = \frac{1}{2}\csc 3(\theta - 45°) + 1$

SOLUTION:

Given $a = \frac{1}{2}$, $b = 3$, $h = 45°$ and $k = 1$.

Period:

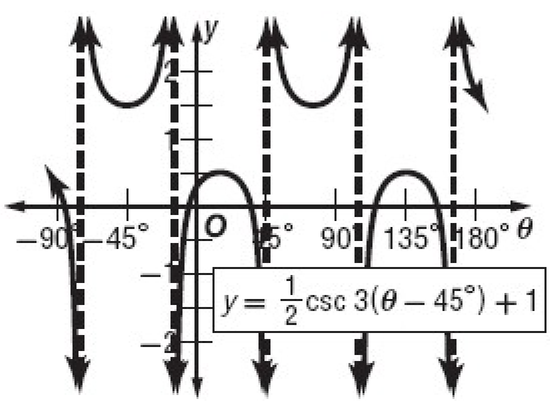
$$\frac{360°}{|b|} = \frac{360°}{|3|}$$
$$= 120°$$

Phase shift: $h = 45°$

Vertical shift: $k = 1$

Midline: $y = 1$

First, graph the midline. Then graph $y = \frac{1}{2}\csc 3(\theta) + 1$ using the midline as reference. Then shift the graph $45°$ to the right.



58. $y = 2\sec\frac{1}{2}(\theta - 90°)$

SOLUTION:

Given a = 2, b = $\frac{1}{2}$ and h = 90°.

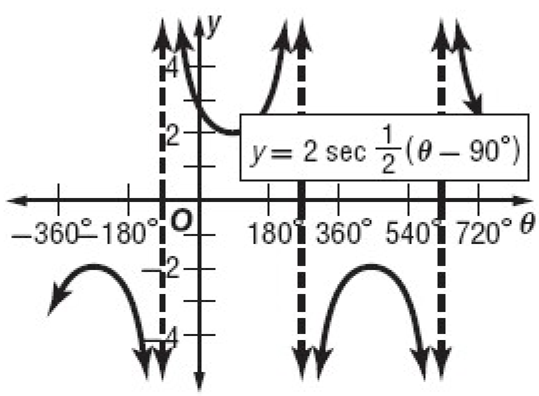
Period:

$$\frac{360°}{|b|} = \frac{360°}{\left|\frac{1}{2}\right|}$$
$$= 720°$$

Phase shift: $h = 90°$

Vertical shift: No vertical shift

Graph $y = 2\sec\frac{1}{2}(\theta)$. Then shift the graph 90° to the right.



59. $y = 4\sec 2\left(\theta + \frac{\pi}{2}\right) - 3$

SOLUTION:

Given a = 4, b = 2, h = $-\frac{\pi}{2}$ and k = –3.

Period:

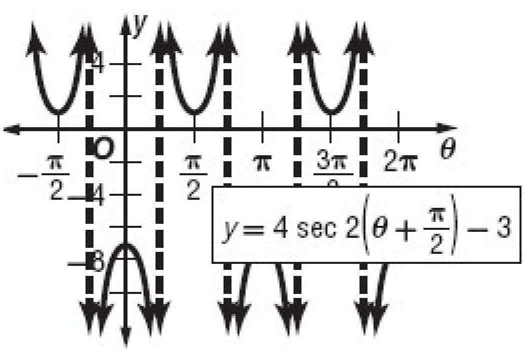
$$\frac{2\pi}{|b|} = \frac{2\pi}{|2|}$$
$$= \pi$$

Phase shift: $h = -\frac{\pi}{2}$

Vertical shift: $k = -3$

Midline: $y = -3$

First, graph the midline. Then graph $y = 4\sec 2\theta - 3$ using the midline as reference. Then shift the graph $\frac{\pi}{2}$ to the left.



60. **CCSS ARGUMENTS** If you are given the amplitude and period of a cosine function, is it *sometimes*, *always*, or *never* possible to find the maximum and minimum values of the function? Explain your reasoning.

SOLUTION:

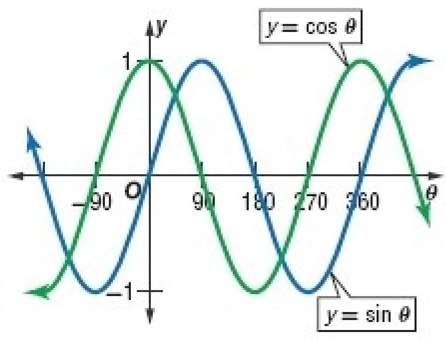
Sometimes; if the function is shifted vertically, then you also need to know the value of the midline. The maximum value is the value of the midline plus the amplitude. The minimum value is the midline value minus the amplitude.

61. **REASONING** Describe how the graph of $y = 3 \sin 2\theta + 1$ is different from $y = \sin \theta$.

SOLUTION:

The graph of $y = 3 \sin 2\theta + 1$ has an amplitude of 3 rather than an amplitude of 1. It is shifted up 1 unit from the parent graph and is compressed so that it has a period of 180°.

62. **WRITING IN MATH** Describe two different phase shifts that will translate the sine curve onto the cosine curve shown at the right. Then write an equation for the new sine curve using each phase shift.



SOLUTION:

When $y = \sin\theta$ is shifted 90° to the left, the sine curve will be translated onto a cosine curve.

Phase shift: $h = -90^\circ$

Therefore, the equation of the curve is $y = \sin\left(\theta - (-90^\circ)\right)$ or $\sin\left(\theta + 90^\circ\right)$

Also, when $y = \sin\theta$ is shifted 270° to the right, the sine curve will be translated onto a cosine curve.

Phase shift: $h = 270^\circ$

Therefore, the equation of the curve is $y = \sin\left(\theta - 270^\circ\right)$.

63. **OPEN ENDED** Write a periodic function that has an amplitude of 2 and midline at $y = -3$. Then graph the function.

SOLUTION:

Sample answer:

The sine function involving phase shifts and vertical shifts is $y = a \sin b(\theta - h) + k$.

Here $a = 2, b = 1, h = 0, k = -3$.

Therefore the equation is $y = 2\sin\theta - 3$.

Amplitude:

$$\begin{aligned} |a| &= |2| \\ &= 1 \end{aligned}$$

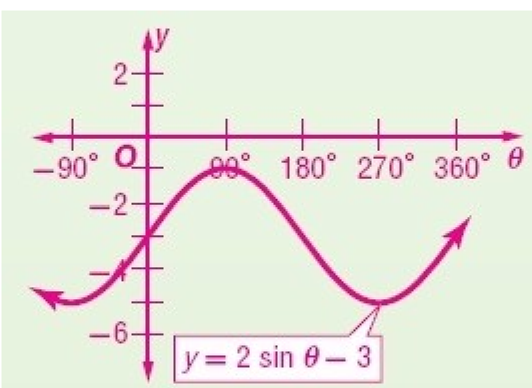
Period:

$$\begin{aligned} \frac{360^\circ}{|b|} &= \frac{360^\circ}{|1|} \\ &= 360^\circ \end{aligned}$$

Vertical shift: $k = -3$

Midline: $y = -3$

To graph $y = 2\sin\theta - 3$, first draw the midline. Then use it to graph $y = 2\sin\theta$ shifted 3 units down.



64. **REASONING** How many different sine graphs pass through the origin $(n\pi, 0)$? Explain your reasoning.

SOLUTION:

Infinitely many; any change in amplitude will create a different graph that has the same θ-intercepts.

65. **GRIDDED RESPONSE** The expression $\frac{3x-1}{4}+\frac{x+6}{4}$ is how much greater than x?

SOLUTION:

Let $y=\frac{3x-1}{4}+\frac{x+6}{4}$.

$$\begin{aligned} y &= \frac{3x-1}{4}+\frac{x+6}{4} \\ &= \frac{4x+5}{4} \\ y &= x+\frac{5}{4} \\ y &= x+1.25 \end{aligned}$$

Therefore, $\frac{3x-1}{4}+\frac{x+6}{4}$ is 1.25 greater than x.

66. Expand $(a-b)^4$.

A a^4-b^4

B $a^4-4ab+b^4$

C $a^4+4a^3b+6a^2b^2+4ab^3+b^4$

D $a^4-4a^3b+6a^2b^2-4ab^3+b^4$

SOLUTION:

$$\begin{aligned} (a-b)^4 &= \frac{4!}{0!(4-0)!}a^4b^0-\frac{4!}{1!(4-1)!}a^3b^1+\cdots+\frac{4!}{4!(4-4)!}a^0b^4 \\ &= a^4-4a^3b+6a^2b^2-4ab^3+b^4 \end{aligned}$$

Therefore, the correct option is D.

67. Solve $\sqrt{x-3}+\sqrt{x+2}=5$.

F 7

G 0,7

H 7, 13

J no solution

SOLUTION:

$$\sqrt{x-3}+\sqrt{x+2}=5$$
$$\sqrt{x-3}=5-\sqrt{x+2}$$

Square on both the sides.

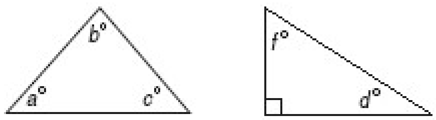
$$x-3=25+x+2-10\sqrt{x+2}$$
$$-3=27-10\sqrt{x+2}$$
$$-30=-10\sqrt{x+2}$$
$$-3=\sqrt{x+2}$$

Again, square on both the sides.

$$(-3)^2=\left(\sqrt{x+2}\right)^2$$
$$9=x+2$$
$$x=7$$

Therefore, the option is F.

68. **GEOMETRY** Using the figures below, what is the average of a, b, c, d, and f?



A 21

B 45

C 50

D 54

SOLUTION:

Since the sum of the measures of the triangle is 180°, we have $a^\circ + b^\circ + c^\circ = 180^\circ$ and $90^\circ + f^\circ + d^\circ = 180^\circ$ or $f^\circ + d^\circ = 90^\circ$

$$\begin{aligned}\text{The average} &= \frac{a^\circ + b^\circ + c^\circ + d^\circ + f^\circ}{5}\\ &= \frac{180^\circ + 90^\circ}{5}\\ &= \frac{270^\circ}{5}\\ &= 54^\circ\end{aligned}$$

Therefore, the correct option is D.

Find the amplitude and period of each function. Then graph the function.

69. $y = 2 \cos \theta$

SOLUTION:

Given $a = 2$ and $b = 1$

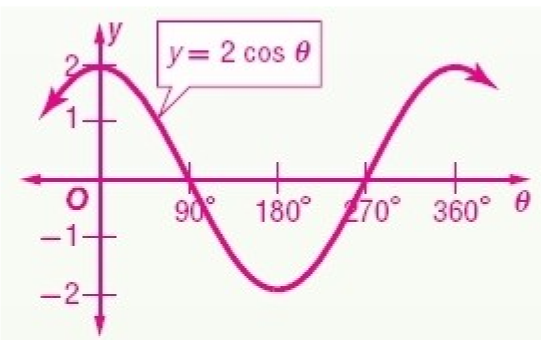
Amplitude:

$$|a| = |2| \\ = 2$$

Period:

$$\frac{360^\circ}{|b|} = \frac{360^\circ}{|1|} \\ = 360^\circ$$

Graph the function $y = 2\cos\theta$.



70. $y = 3 \sin \theta$

SOLUTION:

Given $a = 3$ and $b = 1$

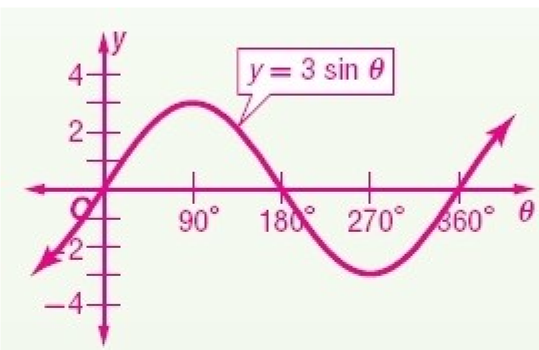
Amplitude:

$$|a| = |3|$$
$$= 3$$

Period:

$$\frac{360^\circ}{|b|} = \frac{360^\circ}{|1|}$$
$$= 360^\circ$$

Graph the function $y = 3\sin\theta$.



71. $y = \sin 2\theta$

SOLUTION:

Given $a = 1$ and $b = 2$

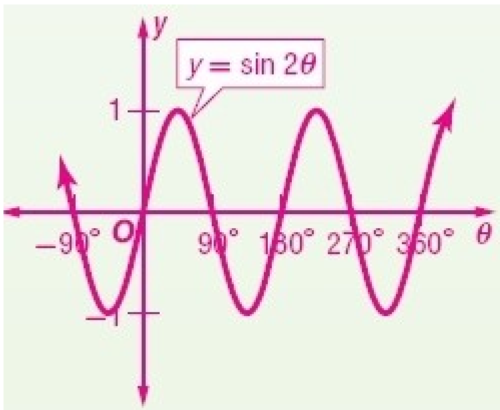
Amplitude:

$$|a| = |1|$$
$$= 1$$

Period:

$$\frac{360^\circ}{|b|} = \frac{360^\circ}{|2|}$$
$$= 180^\circ$$

Graph the function $y = \sin 2\theta$.



Find the exact value of each expression.

72. $\sin\frac{4\pi}{3}$

SOLUTION:

The terminal side of $\frac{4\pi}{3}$ lies in Quadrant III.

Find the measure of the reference angle.

$$\begin{aligned}\theta' &= \theta - \pi \\ &= \frac{4\pi}{3} - \pi \\ &= \frac{\pi}{3}\end{aligned}$$

The sine function is negative in quadrant III.

$$\begin{aligned}\sin\frac{4\pi}{3} &= -\sin\left(\frac{\pi}{3}\right) \\ &= -\frac{\sqrt{3}}{2}\end{aligned}$$

73. $\sin(-30°)$

SOLUTION:

$$\begin{aligned}\sin\left(-30^\circ\right) &= -\sin\left(30^\circ\right) \\ &= -\frac{1}{2}\end{aligned}$$

74. $\cos 405°$

SOLUTION:

$$\begin{aligned}\cos\left(405^\circ\right) &= \cos\left(45^\circ + 360^\circ\right) \\ &= \cos\left(45^\circ\right) \\ &= \frac{1}{\sqrt{2}} \text{ or } \frac{\sqrt{2}}{2}\end{aligned}$$

Determine whether each situation describes a *survey*, an *experiment*, or an *observational study*. Then identify the sample, and suggest a population from which it may have been selected.

75. A group of 220 adults is randomly split into two groups. One group exercises for an hour a day and the other group does not. The body mass indexes are then compared.

SOLUTION:

experiment; sample: people that exercise for an hour a day; population: all adults

76. A soccer coach randomly selects some of his players and gives them a questionnaire asking about their daily sleeping habits.

SOLUTION:

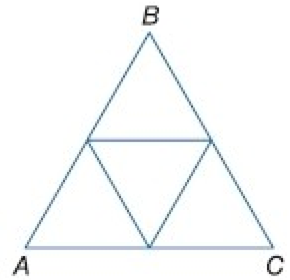
survey; sample: players that received the questionnaire; population: all soccer players

77. A teacher randomly selects 100 students who have part-time jobs and compares their grades.

SOLUTION:

observational study; sample: 100 students selected; population: all students that have part-time jobs

78. **GEOMETRY** Equilateral triangle *ABC* has a perimeter of 39 centimeters. If the midpoints of the sides are connected, a smaller equilateral triangle results. Suppose the process of connecting midpoints of sides and drawing new triangles is continued indefinitely.



a. Write an infinite geometric series to represent the sum of the perimeters of all of the triangles.

b. Find the sum of the perimeters of all of the triangles.

SOLUTION:

a. The perimeter of the equilateral triangle *ABC* is 39 centimeters. Since an equilateral triangle is formed by joining the midpoints, the perimeter of that triangle will be half of the equilateral triangle *ABC*. If the process is repeated again and again, then the perimeter of each equilateral triangle will be half the perimeter of the previous one. Therefore the sum of the perimeters of all of the triangles is $39 + 19.5 + 9.75 + \ldots$

b. Find the value of r.

$$\begin{aligned} r &= \frac{19.5}{39} \\ &= 0.5 \end{aligned}$$

Since $|0.5| < 1$, the sum exists.
Find the sum.

$$s = \frac{a_1}{1 - r}$$

Substitute 39 for a_1, and 0.5 for r.

$$\begin{aligned} s &= \frac{39}{1 - 0.5} \\ &= \frac{39}{0.5} \\ &= 78 \end{aligned}$$

Therefore, the sum of the perimeters of all the triangles is 78 cm.

79. **CONSTRUCTION** A construction company will be fined for each day it is late completing a bridge. The daily fine will be $4000 for the first day and will increase by $1000 each day. Based on its budget, the company can only afford $60,000 in total fines. What is the maximum number of days it can be late?

SOLUTION:

The fine is $4000 for the first day, $5000 for the second day, $6000 for the third day and so on.
Therefore the arithmetic sequence is

$$4,000 + 5,000 + 6,000 + \ldots$$

$$a_1 = 4,000$$
$$d = 1,000$$

To find n:

$$S_n = \frac{n}{2}\left(2a_1 + (n-1)d\right)$$

Substitute 60,000 for S_n, 4,000 for a_1, and 1000 for d.

$$\begin{aligned} 60,000 &= \frac{n}{2}\left(2\left(4,000\right) + (n-1)1000\right) \\ &= \frac{n}{2}\left(7000 + 1000n\right) \\ 60 &= \frac{n}{2}(7 + n) \\ n^2 + 7n - 120 &= 0 \\ (n-8)(n+15) &= 0 \\ n = 8 \text{ or } n &= -15 \end{aligned}$$

Since the number of days cannot be negative, the number of days it can delay is 8 days.

Find each value of θ. Round to the nearest degree.

80. $\sin\theta = \frac{7}{8}$

SOLUTION:

$$\sin\theta = \frac{7}{8}$$
$$\theta = \sin^{-1}\left(\frac{7}{8}\right)$$

Use a calculator.

Keystrokes: 2nd [SIN^{-1}] 7 ÷ 8) ENTER

Output: 61.04497563

Therefore, $\theta \approx 61^\circ$.

81. $\tan\theta = \frac{9}{10}$

SOLUTION:

$$\tan\theta = \frac{9}{10}$$
$$\theta = \tan^{-1}\left(\frac{9}{10}\right)$$

Use a calculator.

Keystrokes: 2nd [TAN^{-1}] 9 ÷ 10) ENTER

Output: 41.9872125

Therefore, $\theta \approx 42^\circ$.

82. $\cos\theta = \frac{1}{4}$

SOLUTION:

$$\cos\theta = \frac{1}{4}$$
$$\theta = \cos^{-1}\left(\frac{1}{4}\right)$$

Use a calculator.

Keystrokes: 2nd [COS^{-1}] 1 ÷ 4) ENTER

Output: 75.52248781

Therefore, $\theta \approx 76^\circ$.

83. $\cos\theta = \frac{4}{5}$

SOLUTION:

$$\cos\theta = \frac{4}{5}$$
$$\theta = \cos^{-1}\left(\frac{4}{5}\right)$$

Use a calculator.

Keystrokes: 2nd [COS^{-1}] 4 ÷ 5) ENTER

Output: 36.86989765

Therefore, $\theta \approx 37^\circ$.

84. $\sin\theta = \frac{5}{6}$

SOLUTION:

$$\sin\theta = \frac{5}{6}$$
$$\theta = \sin^{-1}\left(\frac{5}{6}\right)$$

Use a calculator.

Keystrokes: 2nd [SIN^{-1}] 5 ÷ 6) ENTER

Output: 56.44269024

Therefore, $\theta \approx 56°$.

85. $\tan\theta = \frac{2}{7}$

SOLUTION:

$$\tan\theta = \frac{2}{7}$$
$$\theta = \tan^{-1}\left(\frac{2}{7}\right)$$

Use a calculator.

Keystrokes: 2nd [TAN^{-1}] 2 ÷ 7) ENTER

Output: 15.9453959

Therefore, $\theta \approx 16°$.